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BRIEF COMMUNICATION

THE TRANSVERSE MIGRATION OF BUBBLES INFLUENCED BY WALLS IN VERTICAL BUBBLY FLOW

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INTRODUCTION

Many observations of the peak void fraction values near the duct wall point on the bubble transverse migration from the core region toward the wall at certain bubbly flow regime. Several mechanisms have been proposed to explain this phenomenon: the Magnus and Zhukovski forces acting on the bubbles considered as spinning rigid spheres (Kazin 1964), the influence of the liquid velocity gradient (Lackme 1967, Kobayasi *et al.* 1970), the static pressure change over the channel cross-section due to liquid turbulence (Subbotin *et al.* 1971), the Magnus force due to the circulation of liquid around the bubble together with the Bernoulli force due to the stronger back flow on the wall side of the bubble (Wallis & Richter 1973). These inferences were mostly made from experiments with air–water bubbly mixtures at water volumetric fluxes from approx. 30 to 100 cm/s. On the other hand Rouhani (1976) proposed the hypothesis of rolling vortices generation near the walls as a theoretical explanation for the wall void peaking.

In this communication, the non-equilbrium bubble transverse migration is treated by combining the bubble dispersion and the migration due to the circulation of liquid around the bubble caused by the liquid velocity gradient. The experimental and the theoretical results for the single air bubbles suspended in tap water stream from a point source are presented within the water volumetric flux range from 24.4 to 74.8 cm/s (corresponding bulk water Reynolds numbers from approx. 5000 to 16,000).

EXPERIMENTAL

The experimental facility is schematically presented in figure 1 and explained in detail by Žun (1975). It is similar to the one earlier used by Wallis & Richter (1973), who designed also a special water inlet chamber to reduce secondary flows and the turbulence entrance effects.

Two He–Ne lasers were used to count bubble arrivals in the z–x plane of figure 2. The

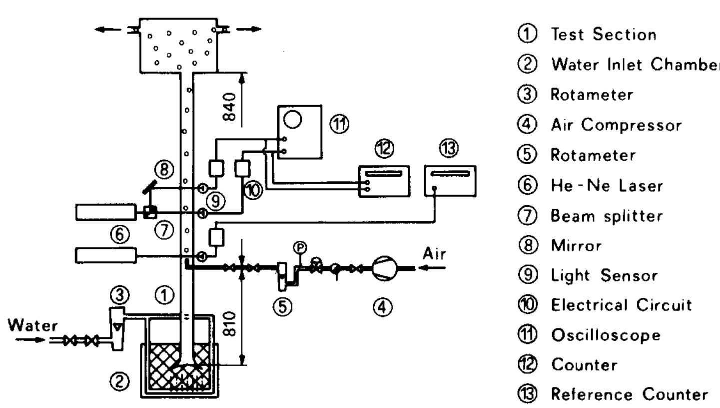

Figure 1. Experimental facility.

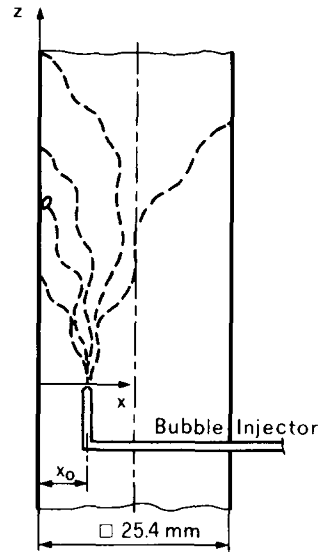

Figure 2. The test section.

measurement lattice points were determined considering the laser beam diameter, the appropriate signal trigger level, and the conserved total number of suspended bubbles. Such integral method required single bubbles and square duct cross-section. To insure the reproducibility of the experiment, samples per 200 suspended bubbles were observed, and the median count rate/sample was taken as a local bubble count rate.

To evaluate the influence of the remaining secondary flows, the bubbles were suspended very close to the wall and their migration toward the duct corners were measured. This phenomenon was evident only at the bulk water Reynolds numbers Re_L smaller than 2200, while the significant bubble transverse migration from the core region toward the wall started at much higher Re_L, approx. 5000.

The range of the water volumetric fluxes j_L ran 24.4–74.8 cm/s ($\mathrm{Re}_L \approx 5000$–16,000). The bubble arrival distributions were measured also in quiescent water. The range of the air bubble equivalent spherical radii was between 0.32 and 2.37 mm.

THEORY

In the presence of significant forces due to gravitational field, drag, liquid velocity field, vortex shedding, and bulk liquid turbulence eddies affecting the bubble motion, a simple equation for the total single bubble flux $\mathbf{j}_b$ was proposed (Žun 1975) which in Cartesian coordinates reads as:

$$\mathbf{j}_b = c_b \mathbf{v}_L + (0, 0, c_b v_\infty) - c_b C_T \frac{\rho_L}{g\Delta\rho} v_{bL}^2 \left(\frac{\partial v_L}{\partial x}, \frac{\partial v_L}{\partial y}, 0\right) - D_b\left(\frac{\partial c_b}{\partial x}, \frac{\partial c_b}{\partial y}, 0\right). \qquad [1]$$

The four terms on the r.h.s. represent, respectively:

(1) The convective bubble flux with c_b being "bubble concentration" or the time-averaged bubble density, and $\mathbf{v}_L$ the liquid velocity,

(2) The contribution by the gravitational field over the terminal bubble velocity v_∞,

(3) The bubble transverse migration terms whose structure is explained below,

(4) The bubble lateral dispersion term, where D_b is the bubble dispersion coefficient.

The bubble transverse migration is conjectured to be due to the "shear" lift and is described by the transverse lift force

$$\mathbf{F}_T = C_T\rho_L(\nabla \times \mathbf{v}_L) \times \mathbf{v}_{bL}. \qquad [2]$$

Here, C_T is the bubble transverse lift coefficient which accounts for the net circulation of liquid around the ellipsoidal bubble, and $\mathbf{v}_{bL}$ is the bubble relative velocity. Furthermore it is assumed that the resultant force acting on the bubble is a composite of forces which are characteristical for the non-distorted (spherical) bubble, and for the bubble distortion and the bulk liquid turbulence. Since the buoyant force $g\nabla\rho$ is proportional to the bubble relative velocity for the non-distorted bubble, while the bubble distortion causes some non-linear dipendence, this is in view of the net bubble motion taken into account by the fourth term in [1]. Hence, the bubble transverse migration term becomes

$$- c_b \frac{v_{bL}}{g\nabla\rho} \mathbf{F}_T. \qquad [3]$$

The idea to utilize the "shear" lift is taken from Lawler & Paul-Chang Lu (1971), where the radial migration of solid particles was described. However, the Magnus effect, which was used by them as well, should not be included here in the case of bubbly flow, because a spinning bubble interface is not expected.

DISCUSSION OF RESULTS

Equation [1] was solved numerically for different values of D_b and C_T using a computer program by Wallis (1975). The following suppositions were made:

(1) The liquid velocity profile is given by the 1/7th power law.

(2) The bubble relative velocity is equal to the bubble terminal velocity. The data were taken from Haberman & Morton (1954).

(3) The initial bubble concentration is a delta function with respect to the injection point.

(4) There are no fluxes perpendicular to walls and the total number of bubbles is conserved.

(5) The bubble concentration c_b is defined by the local bubbles rate per unit area divided by the bubble absolute velocity.

In figures 3–6, the numerical solution of [1] and the experimental results are presented in terms of bubble arrival distributions. In figures 5 and 6, the lateral distance from the channel axis is denoted by b, and the square duct width B is 25.4 mm. The relative count rate of bubbles is given in all figures with respect to the total, reference numbers of suspended bubbles.

It is interesting that the bubble dispersion coefficient D_b of 0.35 cm^2/s fits the experimental data very well within the range of water volumetric fluxes j_L from 24.4 to 53.8 cm/s ($\mathrm{Re}_L \approx$ 5000–12,000) for the region 3 (Wallis 1974) bubble sizes, as shown on figure 3, curves a, b, c. Under this condition, and provided that the bubble injection point x_0 is outside the water core region (figure 4, curves a, b) the bubble transverse lift coefficient C_T has a constant value of 0.3.

Bubbles from region 2D, whose intrinsic dispersion is smaller than of those from region 3, were suspended into a water stream of $j_L = 74.8$ cm/s (figure 3, curve d). The value of D_b is here not much smaller than 0.35 cm^2/s of the curves a, b, c—figure 3, and this is due to the stronger water turbulence. This was the highest water flow rate used because of the steady pressure limitations of the apparatus.

The observations of spherical bubbles from region 2 with radii less than 0.5 mm, which behave similarly to solid spheres of equivalent size and move intrinsicly rectilinearly, showed that bubbles do not migrate any more toward the wall in a deterministic fashion (Žun 1979). A similar conclusion was obtained by Hinata *et al.* (1979). There were no trapped bubbles near the wall, as it always happens with the bubbles from region 2D and 3.

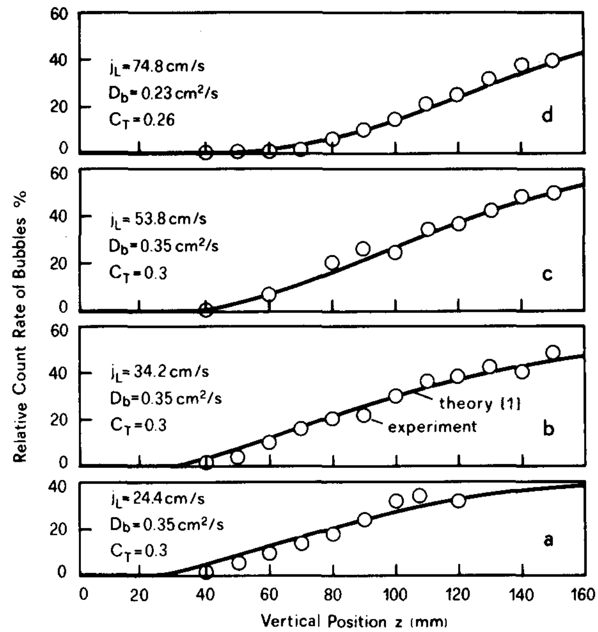

Figure 3. Relative count rate of bubbles vs vertical position z at the distance of 0.5 mm from the duct wall for various water volumetric fluxes j_L. Bubble equivalent sphere radius: a—1 mm, b—0.98 mm, c—0.85 mm, d—0.76 mm. D_b is termed the bubble dispersion coefficient, and C_T is the bubble transverse lift coefficient.

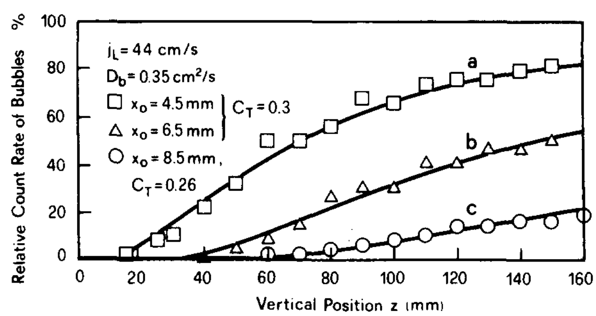

Figure 4. Relative count rate of bubbles vs vertical position z at the distance of 0.5 mm from the duct wall for various bubble injection positions x_0. Bubble equivalent sphere radius is 0.90 mm.

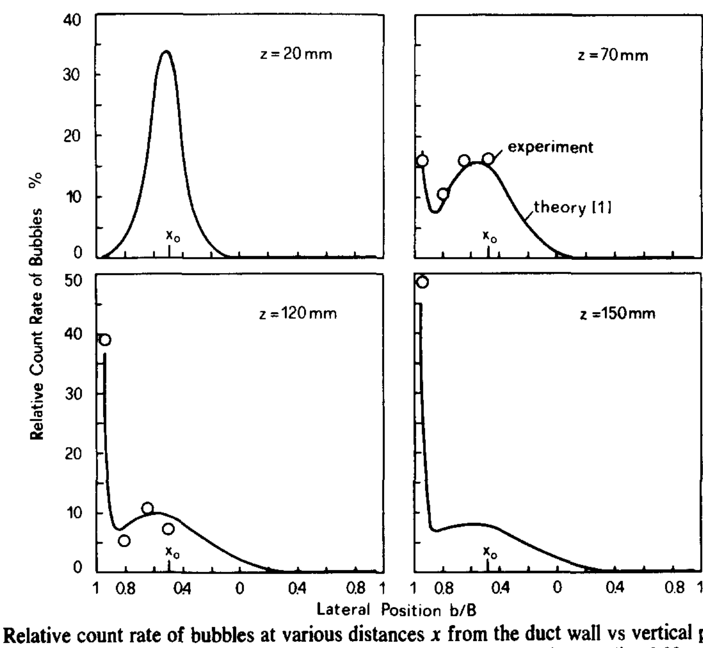

Figure 5. Relative count rate of bubbles at various distances x from the duct wall vs vertical position z for the water volumetric flux 34.2 cm/s and the bubble equivalent sphere radius 0.98 mm.

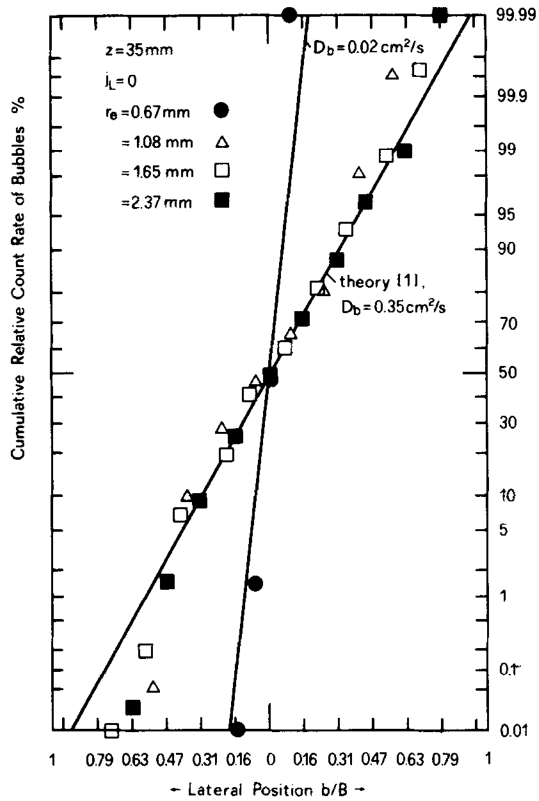

Figure 6(a).

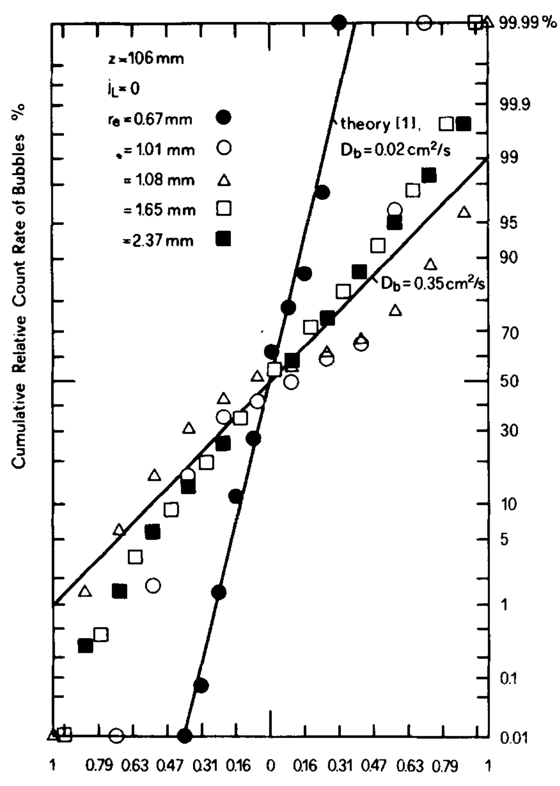

Figure 6(b).

Figure 6(a, b). Cumulative relative count rate of bubbles, across the duct for two different vertical positions *z*, when suspended in quiescent water.

In order to discuss the applicability of the lateral dispersion term in [1], single bubbles from regions 2D, 3 and 4 were suspended in quiescent tap water (Žun 1979). The bubble arrival distributions are presented on probability graph paper in figures 6(a) and 6(b), together with the distributions according to [1] for $C_T = 0$, and $D_b = 0.35\ \mathrm{cm^2/s}$ or $D_b = 0.02\ \mathrm{cm^2/s}$. It is concluded from these figures that bubbles suspended from a point source always have a certain tendency toward lateral dispersion. Single bubbles of regions 2D and 4 disperse almost entirely randomly, while bubbles of region 3, besides some random dispersion, exhibit periodic lateral movement. However, in the turbulent liquid flow (at $\mathrm{Re}_L \gtrsim 5000$ in the presented experiment), a stronger bubble random dispersion is expected, and the use of [1] is then justified.

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